Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. The matrix A satisfies the equation $\begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then: **[1]**

a) $\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$

2. Since α = probability of Type-I error, then 1 - α **[1]**

a) Probability of rejecting H_0 when H_a is true.

b) Probability of not rejecting H_0 when H_0 is true.

c) Probability of not rejecting H_0 when H_a is true.

d) Probability of rejecting H_0 when H_0 is true.

3. Rohan invested ₹300000 in a fund for two years. At the end of two years the investment was worth ₹327000. Rohan's rate of return is **[1]**

a) 6%

b) 7%

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c) 8%

d) 9%

4. The maximum value of the function z = 7x + 5y, subject to constraints $x \leq 3, y \leq 2, x \geq 0, y \geq 0,$ is: **[1]**

a) 31

b) 37

c) 10

d) 21

5. If the matrix $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew-symmetric, then **[1]**

a) x = 2, y = 0

b) x = 2, y = -1

c) x = -2, y = 1

d) x = -2, y = 0

6. The least number of times a fair coin must be tossed so that the probability of getting at least one head is at least 0.8, is **[1]**

a) 3

b) 6

c) 5

d) 7

7. Which one is not a requirement of a binomial distribution? **[1]**

a) There are 2 outcomes for each trial

b) There is a fixed number of trials

c) The probability of success must be the same for all the trials

d) The outcomes must be dependent on each other

8. The order and the degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$, respectively, are **[1]**

a) 3, 2

b) 3, 3

c) 3, $\frac{2}{3}$

d) 3, 1

9. In a 100 m race, A can give B a start of 10 m and C a start of 28 m. How much start can B give to C in the same race? **[1]**

a) 27 m

b) 18 m

c) 20 m

d) 9 m

10. If A is 3×4 matrix and B is a matrix such that A^TB and BA^T are both defined. Then, B is of the type **[1]**

a) 4×3

b) 3×4

c) 4×4

d) 3×3

11. If $x \equiv 4 \pmod 7$, then positive values of x are **[1]**

a) {4, 11, 18, ...}

b) {1, 8, 15, ...}

c) {11, 18, 25, ...}

d) {4, 8, 12, ...}

12. The linear inequality representing the solution set given in figure is **[1]**

a) $|x| \leq 5$

b) $|x| \geq 5$

c) $|x| > 5$

d) $|x| < 5$

13. Two pipes A and B can fill a cistern in 10 minutes and 15 minutes respectively. Pipe C can empty the full cistern in 5 minutes. Pipes A and B are kept open for 4 minutes and then outlet pipe C is also opened. The cistern is **[1]**

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emptied by the output pipe C in

a) 18 minute
b) 10 minute
c) 30 minute
d) 24 minute

14. If the objective function for an L.P.P. is Z = 3x - 4y and the comer points for the bounded feasible region are (0, 0), (5, 0), (6, 5), (6, 8), (4, 10) and (0, 8), then the minimum value of Z occurs at **[1]**

a) (0, 8)
b) (5, 0)
c) (4, 10)
d) (0, 0)

15. If the objective function for an L.P.P. is Z = 3x - 4y and the comer points for the bounded feasible region are (0, 0), (5, 0), (6, 5), (6, 8), (4, 10) and (0, 8), then the maximum of Z occurs at **[1]**

a) (5, 0)
b) (6, 8)
c) (4, 10)
d) (6, 5)

16. In a school, a random sample of 145 students is taken to check whether a student's average calory intake is 1500 or not. The collected data of average calories intake of sample students is presented in a frequency distribution which is called a **[1]**

a) Statistics
b) Parameter
c) Population sampling
d) Sampling distribution

17. $\int \frac{x-1}{\sqrt{x+4}}$ dx is equal to **[1]**

a) $\frac{2}{3}(x+4)^{\frac{3}{2}} - 10\sqrt{x+4} + c$
b) $\sqrt{x+4} + \frac{2}{3}(x+4)^{\frac{3}{2}} + c$
c) $\sqrt{x+4} - \frac{2}{3}(x+4)^{\frac{3}{2}} + c$
d) $\frac{2}{3}(x+4)^{\frac{3}{2}} + 10\sqrt{x+4} + c$

18. Prosperity, Recession, and depression in a business is an example of: **[1]**

a) Irregular Trend
b) Seasonal Trend
c) Cyclical Trend
d) Secular Trend

19. **Assertion (A)**: The matrix A = $\begin{bmatrix} 3 & -1 & 0 \\ \frac{3}{2} & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is rectangular matrix of order 3. **[1]**

Reason (R): If A = $[a_{ij}]_{m\times 1}$, then A is column matrix.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If f(x) = log x, then f"(x) = $-\frac{1}{x^2}$. **[1]**

Reason (R): If y = x^3 log x, then $\frac{d^2y}{dx^2}$ = x(5 + 6 log x).

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Calculate the 3-year moving averages for the loan (in lakh ₹) issued by co-operative banks for farmers in **[2]**

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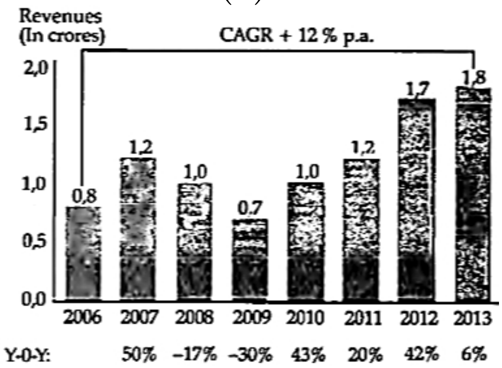
different states of India based on the values given below.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
Loan amount (in lakh ₹)	41.85	40.2	38.12	26.5	55.5	23.6	28.36	33.31	41.1

22. An interviewer gives the following graph on a client's sales in the last 7 years to candidate and said find the CAGR. Given that $\left(\frac{9}{4}\right)^{\frac{1}{7}} = 1.1228$. **[2]**



OR

Suppose a person invested ₹15,000 in a mutual fund and the value of the investment at the time of redemption was ₹25000. If CAGR for this investment is 8.88%. Calculate the number of years for which he has invested the amount.

23. Evaluate: **[2]**

$$\int_0^1 x(1-x)^n \, dx$$

24. If A = diagonal [1, -2, 5], B = diagonal [3, 0, -4] and C = diagonal [-2, 7, 0], then find A + 2B - 3C. **[2]**

OR

If A = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$

25. An 8 litre cylinder contains a mixture of oxygen and nitrogen, the volume of oxygen being 16% of total volume. A few litres of mixture is released and an equal amount of nitrogen is added. The process is repeated twice. As a result, the oxygen content reduces to 9% of total volume. How many litres of mixture is released each time? **[2]**

Section C

26. Solve the initial value problem: $e^{\frac{dy}{dx}} = x + 1$; y(0) = 3 **[3]**

OR

Solve: $(x^2 + 1)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to the initial condition y(0) = 0.

27. Find the purchase price of a ₹600, 8% bond, dividends payable semi-annually redeemable at par in 5 years, if the yield rate is to be 8% compounded semi-annually. **[3]**

28. The marginal cost (MC) of producing x units of a commodity in a day is given as MC = 16x - 1591. The selling price is fixed at ₹9 per unit and the fixed cost is ₹1800 per day. Determine: **[3]**

 i. Cost function
 ii. Revenue function
 iii. Profit function, and
 iv. Maximum profit that can be obtained in a day.

29. In a binomial distribution the sum and product of the mean and the variance are $\frac{25}{3}$ and $\frac{50}{3}$ respectively. Find the distribution. **[3]**

OR

A pair of fair dice is thrown. Let X be the random variable that denotes the minimum of the two numbers which

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appear. Find the probability distribution, mean and variance of X.

30. Calculate trend values from the following data assuming 5-yearly and 7-yearly moving average. **[3]**

Year	1	2	3	4	5	6	7	8
Value	110	104	98	105	109	120	115	110
Year	9	10	11	12	13	14	15	16
Value	114	122	130	127	122	118	130	140

31. The I.Q.'s (intelligence quotients) of 16 students from one area of a city showed a mean of 107 with a standard deviation of 10 while the I.Q.'s of 14 students from another area of the city showed a mean of 112 with a standard deviation of 8. Is there a significant difference between the I.Q.'s of the two groups at **[3]**

i. 1%

ii. 5% level of significance?

Section D

32. A firm can produce three types of cloth, say C_1, C_2, C_3. Three kinds of wool are required for it, say red wool, green wool and blue wool. One unit of length C_1 needs 2 metres of red wool, 3 metres of blue wool; one unit of cloth C_2 needs 3 metres of red wool, 2 metres of green wool and 2 metres of blue wool; and one unit of cloth C_3 needs 5 metres of green wool and 4 metres of blue wool. The firm has only a stock of 16 metres of red wool, 20 metres of green wool and 30 metres of blue wool. It is assumed that the income obtained from one unit of length of cloth is ₹ 6, of cloth C_2 is ₹ 10 and of cloth, C_3 is ₹ 8. Formulate the problem as a linear programming problem to maximize the income. **[5]**

OR

A merchant plans to sell two types of personal computers a desktop model and a portable model that will cost ₹25,000 and ₹40,000 respectively. He estimates that the total monthly demand for computers will not exceed 250 units. Determine the number of units of each type of computer which the merchant should stock to get maximum profit if he does not want to invest more than ₹70 lakhs and his profit on the desktop model is ₹4500 and on the portable model is ₹5000. Make an LPP and solve it graphically.

33. In a kilometre race, if A gives B, a start of 40 metres, then B wins by 19 seconds but if A gives B, a start of 30 seconds then B wins by 40 metres. Find the time taken by each to run a kilometre. **[5]**

34. In a math aptitude test, student scores are found to be normally distributed having mean as 45 and standard deviation 5. What percentage of students have scores **[5]**

i. more than the mean score?

ii. between 30 and 50?

OR

Let X denote the number of hours a Class 12 student studies during a randomly selected school day. The probability that X can take the values x_i, for an unknown constant **k**:

$$P(X=k)=\begin{cases}0\cdot 1 & \text{if} \quad x_i=0\\ kx_i & \text{if} \quad x_i=1 \text{ or } 2\\ k(5-x_i) & \text{if} \quad x_i=3 \text{ or } 4\end{cases}$$

i. Find the value of k.

ii. Determine the probability that the student studied for at least 2 hours.

iii. Determine the probability that the student studied for at most 2 hours.

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35. The cost of a TV depreciates by ₹ 800 during the second year and by ₹ 700 during the third year. Calculate: **[5]**

i. the rate of depreciation per annum.

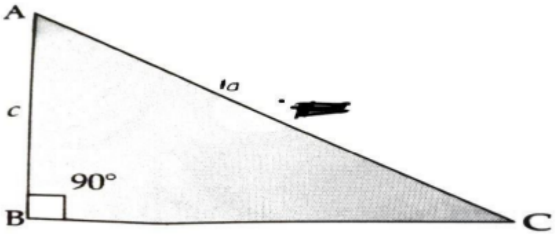
ii. the original cost of the machine.

iii. the value of the TV at the end of third year.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

The sum of the length of hypotenuse and a side of a right-angled triangle is given by AC + BC = 10



(a) Base BC =?

(b) If **S** be the area of the triangle, then find the value of $\frac{dS}{dc}$?

(c) What is the values of c when $\frac{ds}{dc}$ = 0?

OR

Find the value of $\frac{d^2S}{dc^2}$ at C = $\frac{10\sqrt{3}}{3}$?

37. **Read the text carefully and answer the questions:** **[4]**

Flexible payment arrangements, in which the borrower might pay higher sums of his or her choosing, are not the same as EMIs. Borrowers on EMI programmes are usually only allowed to make one set payment per month. Borrowers profit from an EMI since they know exactly how much money they will have to pay towards their loan each month, making personal financial planning easier. Lenders benefit from the loan interest, as it provides a consistent and predictable stream of income.

Example:

A loan of ₹400000 at the interest rate of 6.75% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years.

(Given $(1.005625)^{120} = 1.9603$, $(1.005625)^{60}$ - 1.4001)

(a) Find the size of each monthly payment.

(b) Find the principal outstanding at the beginning of 61st month.

(c) Find the interest paid in 61st payment.

OR

Find the principal contained in 61st payment.

38. A firm produces three products P_1, P_2 and P_3 requiring the mix-up of three materials M_1, M_2 and M_3. The per-unit requirement of each product for each material is as follows: **[4]**

	M_1	M_2	M_3
P_1	2	4	5
P_2	3	2	4
P_3	1	3	2

Using matrix algebra, find:

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i. The total requirement of each material if the firm produces 100, 200 and 300 units of products P_1, P_2 and P_3 respectively.

ii. The per-unit cost of production of each product if the per-unit costs of materials M_1, M_2 and M_3 are ₹10, ₹15 and ₹12 respectively.

iii. The total cost of production.

OR

An amount of ₹5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is ₹358. If the combined income from the first two investments is ₹70 more than the income from the third, find the amount of each investment by matrix method.

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Solution

Section A

1. **(a)** $\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$

   Explanation:

   $\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$

2. **(b)** Probability of not rejecting H_0 when H_0 is true.

   Explanation:

   Probability of not rejecting H_0 when H_0 is true.

3. **(d)** 9%

   Explanation:

   9%

4. **(a)** 31

   Explanation:

   31

5. **(a)** x = 2, y = 0

   Explanation:

   Let A = $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$, then A' = -A

   $\Rightarrow \begin{bmatrix} 0 & 1 & -6 \\ -1 & y & 5 \\ 3x & -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -3x \\ -1 & -y & 5 \\ 6 & -5 & 0 \end{bmatrix}$

   $\Rightarrow$ -3x = -6 $\Rightarrow$ x = 2, y = -y $\Rightarrow$ 2y = 0 $\Rightarrow$ y = 0

   $\therefore$ x = 2, y = 0

   $\therefore$ Option (x = 2, y = 0) is the correct answer.

6. **(a)** 3

   Explanation:

   A fair coin is tossed $\Rightarrow$ p = q $= \frac{1}{2}$

   P(X $\geq$ 1) $\geq$ 0.8

   $\Rightarrow$ 1 - P(0) $\geq$ 0.8

   $\Rightarrow$ P(0) = 0.2

   $\Rightarrow \left(\frac{1}{2}\right)^n$ = 0.2

   $\Rightarrow$ 2^{-n} = 0.2

   $\Rightarrow$ $2^n \geq 5$

   $\Rightarrow$ n $\geq$ 3

7. **(d)** The outcomes must be dependent on each other

   Explanation:

   We know that, in a Binomial distribution,

   i. There are 2 outcomes of each trial.

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ii. There is a fixed number of trails.

iii. The probability of success must be the same for all the trails.

8.

(b) 3, 3

Explanation:

The given differential equation can be written as

$\left(1+3\frac{dy}{dx}\right)^2 = 64\left(\frac{d^3y}{dx^3}\right)^3$

Order = 3, degree = 3

9.

(c) 20 m

Explanation:

A : B = 100 : 90

A : C = 100 : 72

B : C = $\frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{72} = \frac{90}{72}$

When B runs 90 m, C runs 72 m.

When B runs 100 m, C runs $\left(\frac{72}{90} \times 100\right)$m = 80 m.

$\therefore$ B can give C 20 m.

10.

(b) 3×4

Explanation:

We have to find:

Order of A = 3×4

Order of A' = 4×3

As A^TB and BA^T are both defined, so the number of columns in B should be equal to the number of rows in A' for BA' and also the number of columns in A' should be equal to the number of rows in A' for BA'.

Therefore, the order of matrix B is 3×4.

11. **(a)** {4, 11, 18, ...}

Explanation:

x - 4 = $7\lambda \Rightarrow$ x = 4 + 7λ, $\lambda \in$ I

Putting λ = 0, 1, 2, 3, we get x = 4, 11, 18, ...

12.

(b) |x| $\geq$ 5

Explanation:

The given figure is highlighted between $-\infty$ to 5 and 5 to ∞

So, $x \in (-\infty, 5][5, \infty)$

$\Rightarrow$ x $\leq$ -5 and x $\geq$ 5

$\Rightarrow$ |x| $\geq$ 5

13.

(b) 10 minute

Explanation:

part of cistern filled in 5 min

= $5 \times (\frac{1}{10} + \frac{1}{15})$

= $5 \times (\frac{3+2}{30})$

= $5 \times \frac{5}{30}$

= $\frac{5}{6}$ part

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part emptied in 1 min, when all pipes are opened = $\frac{1}{4} - (\frac{1}{10} + \frac{1}{15})$

= $\frac{1}{4} - (\frac{3+2}{30})$

= $\frac{1}{4} - \frac{5}{30}$

= $\frac{15-10}{60}$

= $\frac{5}{60} = \frac{1}{12}$

Now, $\frac{1}{12}$ part is emptied in 1 min

$\therefore$ $\frac{5}{6}$ part is emptied in $12 \times \frac{5}{6}$ = 10 min

14. **(a)** (0, 8)

Explanation:

The values of Z = 3x - 4y at points (0, 0), (5, 0), (6,5), (6,8), (4, 10) and (0, 8) are 0, 15, - 2, -14, -28 and -32 respectively.

Hence, minimum value of Z = - 32 occurs at (0, 8).

15. **(a)** (5, 0)

Explanation:

The values of Z = 3x - 4y at points (0, 0), (5, 0), (6, 5), (6, 8), (4, 10) and (0, 8) are 0, 15, - 2, -14, - 28 and - 32 respectively.

Hence the maximum value of Z = 15 occurs at (5, 0).

16.

(d) Sampling distribution

Explanation:

Sampling distribution

17. **(a)** $\frac{2}{3}(\mathrm{x}+4)^{\frac{3}{2}} - 10\sqrt{\mathrm{x}+4} + \mathrm{c}$

Explanation:

$\int \frac{x-1}{\sqrt{x+4}} dx = \int \frac{x+4-5}{\sqrt{x+4}} dx$

$= \int \sqrt{x+4} dx - 5 \int \frac{1}{\sqrt{x+4}} dx$

$= \frac{2}{3}(x+4)^{3/2} - 10\sqrt{(x+4)} + C$

18.

(c) Cyclical Trend

Explanation:

There are 4 phases through which trade cycles are passed. They are prosperity, recession, depression, and recovery. In economic terms, these 4 stages are called economic fluctuations.

19.

(d) A is false but R is true.

Explanation:

Assertion: A = $\begin{bmatrix} 3 & -1 & 0 \\ \frac{3}{2} & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is a square matrix of order 3.

Reason: In general, A = $[a_{ij}]_{m \times 1}$ is a column matrix.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Assertion: Let y = log x

On differentiating twice w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{x}$

and $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

Reason: Let y = x^3 log x

On differentiating twice w.r.t. x, we get

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$\frac{dy}{dx} = \frac{d}{dx}$ (x^3 log x)

= $x^3 \frac{d}{dx}$ (log x) + log x $\frac{d}{dx}(x^3)$

= $x^3 \left(\frac{1}{x}\right)$ + (log x) ($3x^2$)

= x^2 (1 + 3 log x) [using product rule]

and $\frac{d^2y}{dx^2} = \frac{d}{dx}$ {x^2 (1 + 3 log x)}

= x^2 (0 + $\frac{3}{x}$) + (1 + 3 log x) (2x)

= 3x + 2x (1 + 3 log x)

= x (5 + 6 log x)

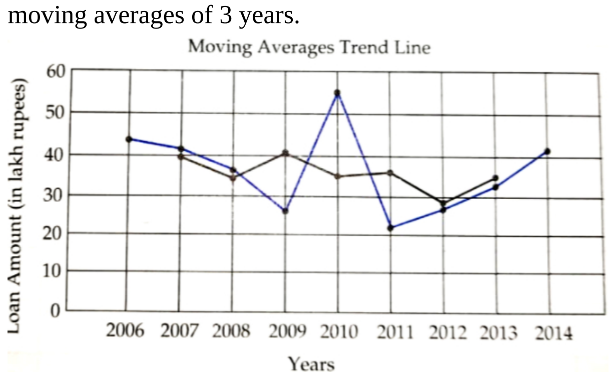
Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Section B

21.

Year	Loan Amount	Three yearly moving totals	Three yearly moving average
2006	41.85	-	-
2007	40.2	120.17	$\frac{120.17}{3}$ = 40.06
2008	38.12	104.82	34.94
2009	26.5	120.12	40.04
2010	55.5	105.6	35.2
2011	23.6	107.46	35.82
2012	28.36	85.27	28.42
2013	33.31	102.77	34.26
2014	41.1	-	-

The graph shows the observation data in pink whereas, the black curve shows the smooth trend curve obtained by calculating moving averages of 3 years.



22. Sales in 2006 were 0.8 crores (beginning value). In 2013, after 7 years, sales increased to 1.8 crores.

CAGR = $\left(\frac{\text{End value}}{\text{Beginning value}}\right)^{\frac{1}{n}} - 1$

= $\left(\frac{1.8}{0.8}\right)^{\frac{1}{7}} - 1 = \left(\frac{9}{4}\right)^{\frac{1}{7}}$ -1

= 0.1228

CAGR% = 12.28%

OR

EV = ₹25000, SV = ₹15000, CAGR = 8.88% n = ?

CAGR = $\left|\left(\frac{EV}{SV}\right)^{\frac{1}{n}} - 1\right| \times 100$

$8.88 = \left[\left(\frac{25,000}{15,000}\right)^{\frac{1}{n}} - 1\right] \times 100$

0.0888 + 1 = $(1.666)^{\frac{1}{n}}$

1.0888 = $(1.666)^{\frac{1}{n}}$

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$\log(1.0888) = \frac{1}{n}\log(1.666)$

$n = \frac{\log(1.666)}{\log(1.0888)} = \frac{0.2216}{0.0369}$ = 6.005 ≈ 6 years

23. $\int_0^1 x(1-x)^n dx = \int_0^1 (1-x)(1-(1-x))^n dx$

$= \int_0^1 (1-x)x^n dx = \int_0^1 \left(x^n - x^{n+1}\right) dx$

$= \left[\frac{x^{n+1}}{n+1}\right]_0^1 - \left[\frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+1}(1-0) - \frac{1}{n+2}(1-0)$

$= \frac{1}{n+1} - \frac{1}{n+2} = \frac{(n+2)-(n+1)}{(n+1)(n+2)} = \frac{1}{n^2+3n+2}$

24. A + 2B - 3C = diagonal [1 + 2 × 3 - 3 (-2), -2 + 2 × 0 - 3 × 7, 5 + 2 × (-4) -3 × 0]

= diagonal [13, -23, -3].

OR

Given, A = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$3A^2 - 2B + I$

$= 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 3\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3-0+1 & -12-8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$

$= \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

25. We have,

Original volume of oxygen = 16% of 8 litre = $\frac{16}{100}$ × 8 = 1.28 litre

Volume of oxygen left after two operations = 9% of 8 litre = $\frac{9}{100}$ × 8 = 0.72 litre

Suppose y litres of mixture is released each time. Then,

$\frac{\text{Volume of oxygen left after two operations}}{\text{Original volume of oxygen}} = \left(1 - \frac{y}{8}\right)^2$

$\Rightarrow \frac{0.72}{1.28} = \left(1 - \frac{y}{8}\right)^2 \Rightarrow \frac{9}{16} = \left(1 - \frac{y}{8}\right)^2$

$\Rightarrow 1 - \frac{y}{8} = \frac{3}{4} \Rightarrow \frac{y}{8} = \frac{1}{4}$

⇒ y = 2

Hence, 2 litres of mixture is released each time.

Section C

26. The given differential equation is,

$e^{\frac{dy}{dx}}$ = x + 1

Taking log on both sides, we get,

$\frac{dy}{dx}$ log e = log (x + 1)

⇒ $\frac{dy}{dx}$ = log (x + 1)

⇒ dy = {log (x + 1)} dx

Integrating both sides, we get

$\int$ dy = $\int$ {log (x + 1) dx

$\Rightarrow y = \int \underset{II}{1} \times \log(\underset{I}{x} + 1)dx$

⇒ y = log (x + 1) $\int$ 1 dx - $\int$[$\frac{d}{dx}$(log x + 1) $\int$ 1dx] dx

⇒ y = x log (x + 1) - $\int \frac{x}{x+1}dx$

⇒ y = x log (x + 1) - $\int \left(1 - \frac{1}{x+1}\right) dx$

⇒ y = x log (x + 1) - x + log (x + 1) + C ...(i)

It is given that y(0) = 3

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$\therefore 3 = 0 \times \log(0 + 1) - 0 + \log(0 + 1) + C$

$\Rightarrow C = 3$

Substituting the value of C in (i), we get

$y = x \log(x + 1) + \log(x + 1) - x + 3$

$\Rightarrow y = (x + 1)\log(x + 1) - x + 3$

Hence, $y = (x + 1)\log(x + 1) - x + 3$ is the solution to the given differential equation.

OR

The given differential equation can be written as

$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where

$P = \frac{2x}{1+x^2}$ and $Q = \frac{4x^2}{1+x^2}$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \frac{2x}{(1+x^2)} dx} = e^{\log(1+x^2)} = 1 + x^2$

Multiplying both sides of (i) by I.F. $= (1 + x^2)$, we get

$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

Integrating both sides with respect to x, we get

$y(1 + x^2) = \int 4x^2\, dx + C$ [Using: $y(\text{I.F.}) = \int Q\,(\text{I.F.})\, dx + C$]

$\Rightarrow y(1 + x^2) = \frac{4x^3}{3} + C$ ...(ii)

It is given that y = 0, when x = 0. Putting x = 0 and y = 0 in (i), we get

$0 = 0 + C \Rightarrow C = 0$

Substituting C = 0 in (ii), we get $y = \frac{4x^3}{3(1+x^2)}$, which is the required solution.

27. Face value of the bond C = ₹600

Nominal rate of interest i = 8% or 0.08

As dividends are paid semi-annually

Therefore, Rate of interest per period $i_d = \frac{0.08}{2} = 0.04$

Therefore, periodic dividend payment $R = C \times i_d = 600 \times 0.04 = 24$

So, semi-annual dividend R is ₹24

Yield rate is 8% = 0.08, compounded semi annually

Therefore $i = \frac{0.08}{2} = 0.04$

No. of years n = 5

Therefore, no. of dividend periods (n) = $5 \times 2 = 10$

Purchase price (V) of the bond is given by

$V = R\left|\frac{1-(1+i)^{-n}}{i}\right| + C(1+i)^{-n}$

$= 24\left|\frac{1-(1+0.04)^{-10}}{0.04}\right| + 600(1+0.04)^{-10}$

$= 24\left|\frac{1-(1.04)^{-10}}{0.04}\right| + 600(1.04)^{-10}$

$= 24\left[\frac{1-0.6755}{0.04}\right| + 600(0.6755)$

= 194.7 + 405.3 = 600

Therefore, purchase price of bond is ₹600.

28. i. Let P, C, and R be the profit function, the cost function and the revenue function respectively. It is given that

MC = 16x - 1591

$\Rightarrow \frac{dC}{dx} = 16x - 1591$

Integrating both sides with respect to x, we obtain

$C = \int(16x - 1591)dx$

$\Rightarrow C = 8x^2 - 1591x + K$ ...(i)

where K is an arbitrary constant. It is given that the fixed cost is ₹ 800 i.e. C = 1800 when x = 0. Substituting these values in (i), we obtain K = 1800. Therefore, the cost function is given by

$C = 8x^2 - 1591x + 1800$

ii. The selling price is fixed at ₹ 9 per unit. So, the revenue function R is given by R = 9x

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iii. the profit function P is given by

P = R - C

$\Rightarrow$ P = 9x - (8x^2 - 1591x + 1800)

$\Rightarrow$ P = -8x^2 + 1600x - 1800

iv. We have,

P = -8x^2 + 1600x - 1800

$\Rightarrow \frac{dP}{dx}$ = -16x + 1600 and $\frac{d^2P}{dx^2}$ = -16

For maximum profit, we must have

$\frac{dP}{dx}$ = 0 $\Rightarrow$ -16x + 1600 = 0 $\Rightarrow$ x = 100

Clearly, $\frac{d^2P}{dx^2}$ = -16 < 0 for all x.

Thus, P is maximum when x = 100. The corresponding profit is given by

P = -8 $\times$ 100^2 + 1600 $\times$ 100 - 1800 = 78,200

Hence, maximum profit = ₹ 78,200

29. We have,

Sum of the mean and variance = $\frac{25}{3}$

$\Rightarrow$ np + npq = $\frac{25}{3}$

$\Rightarrow$ np(1 + q) = $\frac{25}{3}$ ...(i)

Product of the mean and variance = $\frac{50}{3}$

$\Rightarrow$ np(npq) = $\frac{50}{3}$ ...(ii)

Dividing eq. (ii) by eq. (i), we have,

$\frac{\text{np(npq)}}{\text{np(1+q)}} = \frac{50}{3} \times \frac{3}{25}$

$\Rightarrow \frac{npq}{1+q} = 2$

$\Rightarrow$ npq = 2(1 + q)

$\Rightarrow$ np(1 - p) = 2(2 - p)

$\Rightarrow np = \frac{2(2-p)}{(1-p)}$

Substituting this value in np + npq = $\frac{25}{3}$, we have,

$\frac{2(2-p)}{(1-p)}(2-p) = \frac{25}{3}$

$\Rightarrow$ 6(4 - 4p + p^2) = 25 - 25p

$\Rightarrow$ 6p^2 + p - 1 = 0

$\Rightarrow$ (3p - 1) (2p + 1) = 0

$\Rightarrow p = \frac{1}{3}$ or $\frac{-1}{2}$

As p cannot be negative, therefore possible value of p is $\frac{1}{3}$

q = 1 - p = $\frac{2}{3}$

$\Rightarrow$ np + npq = $\frac{25}{3}$

$\Rightarrow n\left(\frac{1}{3}\right)\left(1+\frac{2}{3}\right) = \frac{25}{3}$

$\Rightarrow$ n = 15

$\therefore$ P(X = r) $= {}^{15}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{15-r}$, r = 0, 1, 2 15

OR

Let X be a random variable denoting the event of getting twice the number. Then, X can take the values 1, 2, 3, 4, 5 and 6.

Thus, the probability distribution of X is as follows:

X	1	2	3	4	5	6
P(X)	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

Computation of mean and variance:

x_i	p_i	p_ix_i	$p_ix_i^2$
1	$\frac{11}{36}$	$\frac{11}{36}$	$\frac{11}{36}$
2	$\frac{9}{36}$	$\frac{18}{36}$	1

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3	$\frac{7}{36}$	$\frac{21}{36}$	$\frac{63}{36}$
4	$\frac{5}{36}$	$\frac{20}{36}$	$\frac{80}{36}$
5	$\frac{3}{36}$	$\frac{15}{36}$	$\frac{75}{36}$
6	$\frac{1}{36}$	$\frac{6}{36}$	1
		$\sum p_i x_i = \frac{91}{36} = 2.5$	$\sum p_i x_i^2 = \frac{301}{36} = 8.4$

Therefore,Mean = $\sum p_i x_i$ = 2.5

Variance = $\sum p_i x_i^2$ - (Mean)^2 = 8.4 - 6.25 = 2.15

30.

Year	Value	Moving Totals		Moving Average	
		5 year	**7 year**	**5 year**	**7 year**
1	110	-	-	-	-
2	104	-	-	-	-
3	98	526	-	105.2	-
4	105	536	761	107.2	108.71
5	109	547	761	109.4	108.71
6	120	559	771	111.8	110.14
7	115	568	795	113.6	113.57
8	110	581	820	116.2	117.14
9	114	591	838	118.2	119.71
10	122	603	840	120.6	120.00
11	130	615	843	123.0	120.43
12	127	619	863	123.8	123.29
13	122	627	889	125.4	127.00
15	130	-	-	-	-
16	140	-	-	-	-

31. Performing independent samples t-test,not assuming equal variances.

Assumptions: both populations must be normal.

The null hypothesis: the mean IQs are equal.

The alternative hypothesis: the mean IQs are different.

Degrees of freedom: df= min(N1, N2) - 1 = 13

The standard error:

SE = $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{10^2}{16} + \frac{8^2}{14}}$ = 3.2896

The test statistics:

t = $\frac{\left(\bar{X}_1 - \bar{X}_2\right) - 0}{SE} = \frac{107-112}{3.2896}$ = -1.52

The two-tailed cumulative probability value associated with the given t-statistic can be determined from the Student's t-distribution table or calculated using the technology (function T.DIST.2T() of MS Excel).

For df = 13 and t = -1.52, p = 0.152

Since the p-value is greater than both α values, fail to reject the null hypothesis at both significance levels.

The samples do not provide sufficient evidence to conclude the difference between the mean IQs.

Section D

32. The given information can be put in the following tabular form:

	Cloth C_1	Cloth C_2	Cloth C_3	Total quality of wool available

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Red Wool	2	3	0	16
Green Wool	0	2	5	20
Blue Wool	3	2	4	30
Income (in ₹)	6	10	8	

Let x_1, x_2 and x_3 be the quantity produced in metres of the cloth of type C_1, C_2 and C_3 respectively.

Since 2 metres of red wool are required for one metre of cloth C_1 and x_1 metres of cloth C_1 are produced, therefore $2x_1$ metres of red wool will be required for cloth C_1. Similarly, cloth C_2 requires $3x_2$ metres of red wool and cloth C_3 does not require red wool.

Thus, the total quantity of red wool required is $2x_1 + 3x_2 + 0x_3$.

But, the maximum available quantity of red wool is 16 metres.

$\therefore 2x_1 + 3x_2 + 0x_3 \leq 16$

Similarly, the total quantities of green and blue wool required are

$0x_1 + 2x_2 + 5x_3$ and $3x_1 + 2x_2 + 4x_3$ respectively.

But, the total quantities of green and blue wool available are 20 metres and 30 metres respectively.

$\therefore 0x_1 + 2x_2 + 5x_3 \leq 20$ and $3x_1 + 2x_2 + 4x_3 \leq 30$

Also, we cannot produce negative quantities, therefore

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

The total income is $Z = 6x_1 + 10x_2 + 8x_3$

Hence, the linear programming problem for the given problem is

Maximize $Z = 6x_1 + 10x_2 + 8x_3$

Subject to the constraints

$2x_1 + 3x_2 + 0x_3 \leq 16$

$0x_1 + 2x_2 + 5x_3 \leq 20$

$3x_1 + 2x_2 + 4x_3 \leq 30$

and, $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

OR

Let x and y be the number of desktop models and portable models respectively.

Number of desktop models and portable models cannot be negative.

Therefore, $x \geq 0, y \geq 0$

It is given that the monthly demand will not exist 250 units.

$\therefore x + y \leq 250$

Cost of desktop and portable models is ₹25,000 and ₹40,000 respectively.

Therefore, cost of x desktop model and y portable model is ₹25,000 and ₹40,000 respectively and he does not want to invest more than ₹70 lakhs.

$25000x + 40000y \leq 7000000$

Profit on the desktop model is ₹4500 and on the portable model is ₹5000. Therefore, profit made by x desktop model and y portable model is ₹4500 x and ₹5000 y respectively.

Total profit = $Z = 4500x + 5000y$

The mathematical form of the given LPP is:

Maximize $Z = 4500x + 5000y$

Subject to constraints: $x + y \leq 250$

$25000x + 40000y \leq 7000000$

$x \geq 0, y \geq 0,$

First we will convert inequations into equations as follows:

$x + y = 250$, $25000x + 40000y = 7000000$, $x = 0$ and $y = 0$

Region represented by $x + y = 250$;

The line $x + y = 250$ meets the coordinate axes at A(250, 0) and B(0, 250) respectively. By joining these points we obtain the line $x + y = 250$. Clearly (0, 0) satisfies the $x + y = 250$. So, the region which contains the origin represents the solution set of the inequation $x + y \leq 250$.

Region represented by $25000x + 40000y \leq 7000000$:

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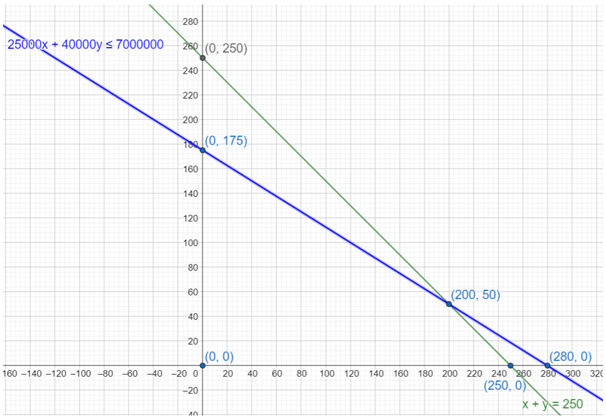
The line 25000 x + 40000 y = 7000000 meets the coordinate axes at C(280, 0) and D(0, 175) respectively. By joining these points we obtain the line 25000 x + 40000 y = 7000000. Clearly (0, 0) satisfies the inequation 25000 x + 40000 y $\leq$ 7000000. So,the region which contains the origin represents the solution set of the inequation 25000 x + 40000 y $\leq$ 7000000

Region represented by x $\geq$ 0 and y $\geq$ 0:

Since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0.

The feasible region determined by the system of constraints x + y $\leq$ 250, 25000 x + 40000y $\leq$ 7000000, x $\geq$ 0 and y $\geq$ 0 are as follows:



The corner points are O(0, 0), D(0, 175), E(200, 50) and A(250, 0)

The values of the objective function Z at corner points of the feasible region are given in the following table:

Corner Points	Z = 4500x + 5000y	
O(0, 0)	0	
D(0, 175)	875000	
E(200, 50)	1150000	$\rightarrow$ Maximum
A(250, 0)	1125000	

Clearly, Z is maximum at x = 200 and y = 50 and the maximum value of Z at this point is 1150000.

Thus, 200 desktop models and 50 portable units should be sold to maximize the profit.

33. Let the time taken by A and B to rim 1 km be x and y seconds respectively. If A gives B, a start of 40 metres, it means that in the same time A runs 1000 metres while B runs (1000 - 40) m = 960 m. Now,

A and B take 125 seconds and 150 seconds respectively to run 1 kilometer

34. Given μ = 45, σ = 5.

Let X denote the test score in the maths aptitude test.

i. P(X > 45) = $P\left(Z > \frac{45-45}{5}\right)$ = P(Z > 0)

= 1 - F(0) = 1 - 0.5

= 0.5 or 50%

ii. P(30 < X < 50) = P$\left(\frac{30-45}{5} < Z < \frac{50-45}{5}\right)$ = P(-3 < Z < 1)

= F(1) - F(-3) = F(1) - [1 - F(3)]

= 0.8413 - 1 + 0.9986 = 0.8399

= 83.99% or 84%

OR

i. 0.1 + k + 2k + 2k + k = 1

$\Rightarrow$ 0.1 + 6k = 1

$\Rightarrow k = \frac{3}{20}$

ii. $P(X \geq 2)$ = P(2) + P(3) + P(4)

= 2k + 2k + k

= 5k = $\frac{3}{4}$

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iii. $P(X \leq 2)$ = P(0) + P(1) + P(2)

= 0.1 + k + 2k

$= \frac{1}{10} + \frac{9}{20} = \frac{11}{20}$

35. i. Let the original cost of the TV be ₹ P and the rate of depredation be r % p.a. Then the value of TV (in ₹) after 1 year, 2 years and 3 years are P(1 - i),

$P(1 - i)^2$ and $P(1 - i)^3$, respectively, where i = $\frac{r}{100}$

According to question,

$P(1 - t) - P(1 - i)^2 = 800$

and $P(1 - i)^2 - P(1 - i)^3 = 700$

$\Rightarrow$ P(1 - i) [1 - (1 - i)] = 800

and $P(1 - i)^2$ [1 - (1 - i)] = 700

$\Rightarrow$P(1 - i)i = 800...(i)

and $P(1 - i)^2 i = 700$...(ii)

On dividing eq. (ii) by eq (i), we get

1 - i = $\frac{700}{800}$

$\Rightarrow$ 1 - i = $\frac{7}{8}$

$\Rightarrow$ i = 1 - $\frac{7}{8}$

$\Rightarrow$ i = $\frac{1}{8}$

$\therefore \frac{r}{100} = \frac{1}{8}$

$\Rightarrow$ r = $\frac{100}{8}$= 12.5%

Hence, the rate of depreciation = 12.5% p.a.

ii. Putting i = $\frac{1}{8}$ in eq. (i), we get

$P\left(1 - \frac{1}{8}\right) \times \frac{1}{8}$ = 800

$\Rightarrow P \times \frac{7}{8 \times 8}$ = 800

P = $\frac{800 \times 8 \times 8}{7}$

= 7314.28

Hence, the original cost of TV is ₹ 7314.

iii. The value of TV at the end of 3^{rd} year = $P(1 - i)^3$

$= 7314\left(1 - \frac{1}{8}\right)^3$

$= 7314\left(\frac{7}{8}\right)^3$

$= \frac{7314 \times 7 \times 7 \times 7}{8 \times 8 \times 8}$

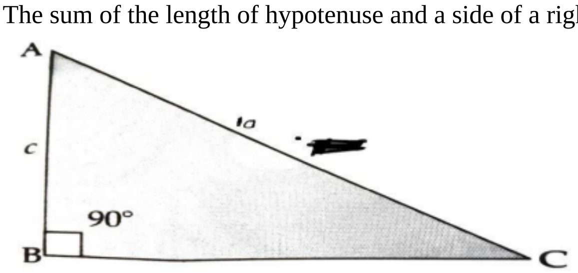
= 4899.80 $\approx$ ₹ 4900

Thus, the value of TV at the end of the 3^{rd} year is ₹ 4900.

Section E

36. **Read the text carefully and answer the questions:**

The sum of the length of hypotenuse and a side of a right-angled triangle is given by AC + BC = 10



(i) $\frac{100-c^2}{20}$

(ii) $\frac{100-3c^2}{40}$

(iii)$\frac{10\sqrt{3}}{3}$

OR

$\frac{-\sqrt{3}}{2}$

37. **Read the text carefully and answer the questions:**

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Flexible payment arrangements, in which the borrower might pay higher sums of his or her choosing, are not the same as EMIs. Borrowers on EMI programmes are usually only allowed to make one set payment per month. Borrowers profit from an EMI since they know exactly how much money they will have to pay towards their loan each month, making personal financial planning easier. Lenders benefit from the loan interest, as it provides a consistent and predictable stream of income.

Example:

A loan of ₹400000 at the interest rate of 6.75% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years.

(Given $(1.005625)^{120} = 1.9603$, $(1.005625)^{60}$ - 1.4001)

(i) ₹ 4593

(ii) ₹ 233336.89

(iii)₹ 1312.52

OR

₹ 3280.48

38. The matrix showing per unit requirement of materials M_1, M_2 and M_3 in producing three products P_1, P_2 and P_3 is

$$A = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \end{array} \begin{array}{c} \begin{array}{ccc} P_2 & P_3 & P_3 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 3 \\ 5 & 4 & 2 \end{bmatrix} \end{array}$$

i. (i) The matrix representing the requirements of products P_1, P_2 and P_3 is

$$B = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

So, the requirements of each material for producing the given quantities of three products is given by the product

$$AB = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \end{array} \begin{array}{c} \begin{array}{ccc} P_2 & P_3 & P_3 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 3 \\ 5 & 4 & 2 \end{bmatrix} \end{array} \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

$$\Rightarrow AB = \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array} \begin{bmatrix} 200 + 600 + 300 \\ 400 + 400 + 900 \\ 500 + 800 + 600 \end{bmatrix} = \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array} \begin{bmatrix} 1100 \\ 1700 \\ 1900 \end{bmatrix}$$

Thus, 1100 units of material M_1, 1700 emits of material M_2 and 1900 units of material M_3 are required to produce 100 units of P_1, 200 units of P_2 and 300 units of P_3.

ii. The matrix representing per-unit costs of materials M_1, M_2 and M_3 is as given below:

$$C = \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array} \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

The matrix exhibiting the materials M_1, M_2 and M_3 in three products P_1, P_2 and P_3 is

$$D = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} 2 & 4 & 5 \\ 3 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix} \end{array}$$

So, the per emit cost of each product is given by the matrix product

$$DC = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} 2 & 4 & 5 \\ 3 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix} \end{array} \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array} \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix} = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \begin{bmatrix} 20 + 60 + 60 \\ 30 + 30 + 48 \\ 10 + 45 + 24 \end{bmatrix} = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \begin{bmatrix} 140 \\ 108 \\ 79 \end{bmatrix}$$

Hence, per unit cost of production of products P_1, P_2 and P_3 are ₹ 140, ₹ 180 and ₹ 79 respectively.

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iii. The total cost of product is given by the matrix product

$$\begin{array}{ccc} P_1 & P_2 & P_3 \end{array} \\ \begin{bmatrix} 100 & 200 & 300 \end{bmatrix} \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}\begin{bmatrix} 140 \\ 108 \\ 79 \end{bmatrix} = (14,000 + 21,600 + 23,700) = ₹\ 59,300$$

Hence, the total cost of product is ₹ 59,300

OR

Let x, y and z ₹ be the investments at the rates of interest of 6%, 7% and 8% per annum respectively. Then,

Total investment = ₹5000

$\Rightarrow$ x + y + z = 5000

Now, Income from first investment of ₹ x = ₹ $\frac{6x}{100}$

Income from second investment of ₹ y = ₹ $\frac{7y}{100}$

Income from third investment of ₹ z = ₹ $\frac{8z}{100}$

$\therefore$ Total annual income = ₹ $\left(\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100}\right)$

$\Rightarrow \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100}$ = 358 [$\because$ Total annual income = ₹358]

It is given that the combined income from the first two investments is ₹70 more than the income from the third

$\therefore \frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100} \Rightarrow$ 6x + 7y - 8z = 7000

Thus, we obtain the following system of simultaneous linear equations:

x + y + z = 5000

6x + 7y + 8z = 35800

6x + 7y - 8z = 7000

This system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

or, AX = B, where A = $\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$

Now, |A| = $\begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$ = 1 (-56 - 56) - (-48 - 48) + (42 - 42) = -16 $\neq$ 0

So, A^{-1} exists and the solution of the given system of equations is given by X = A^{-1} B

Let C_{ij} be the cofactor of a_{ij} in A = $[a_{ij}]$. Then,

C_{11} = -112, C_{12} = 96, C_{13} = 0, C_{21} = 15, C_{22} = -14,

C_{23} = -1, C_{31} = 1, C_{32} = -2 and C_{33} = 1

$\therefore$ adj A = $\begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

So, A^{-1} = $\frac{1}{|A|}$ (adj A) = $-\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

Hence, the solution is given by

X = A^{-1} B = $-\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -560000 & +537000 & +7000 \\ 480000 & -501200 & -14000 \\ 0 & -35800 & +7000 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$$

$\Rightarrow$ x = 1000, y = 2200 and z = 1800

Hence, three investments are of ₹1000, ₹2200 and ₹1800 respectively.

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